

Committee Size and Incentives in Political Selection^{*}

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Abstract

This paper studies committee-based selection when a winner is chosen by majority vote among committee members who make independent, noisy assessments of competing candidates. Each candidate chooses a costly investment that raises the probability of support from any given committee member. The main insight is a pivotality effect: marginal returns to investment arise only in states where the committee is evenly split, so that one member's decision can change the final outcome. As committee size increases, such pivotal states become less likely, which reduces the incentive to invest. At the same time, larger committees aggregate information more effectively, increasing the probability that the candidate with the highest investment is selected. These two forces generate a non-monotonic relationship between committee size and aggregate investment, and result in an interior committee size that maximizes candidates' investment.

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1 Introduction

Many important political and economic allocations are determined not by a single decision maker, but by a committee. Party organizations rely on collective bodies to select candidates, public agencies use panels to award grants and procurement contracts, universities use committees to make hiring and promotion decisions, and many regulatory and judicial institutions delegate decisions to groups rather than individuals. In all of these settings, institutional design shapes not only who is selected, but also the incentives candidates face to invest resources (whether money, time, or other forms of investment) in order to be selected in the first place. Our central question, therefore, is how committee-based selection affects those incentives.

Consider a committee-based selection process where two candidates compete for a position by choosing costly investments. After observing those investments imperfectly, each committee member independently supports one of the candidates, and the winner is the candidate who receives the support of a majority of the committee.

The main insight of the paper is that the committee, as opposed to a single arbiter, generates a pivotality effect. A marginal increase in a candidate's investment affects that candidate's payoff only in states in which the committee is sufficiently close for one member's decision to change the final outcome. To see the intuition, consider a committee of three members. If a candidate is already supported by two members, a marginal increase in investment has no first-order effect on the final outcome, since the candidate has already secured the prize. If the candidate is supported by none, a marginal increase also has negligible value, since it would have to overturn more than one decision. It is only when the committee is split one-to-one and the remaining member is decisive that a small increase in investment has a first-order effect on the probability of winning. More generally, the return to marginal investment is tied to the probability that the committee is evenly split.

This mechanism has an immediate implication for institutional design. As committee size increases, the probability of a close vote falls, so the set of states in which an individual member is pivotal becomes less likely. This reduces the marginal return

to investment and reduces candidates' incentives to invest. At the same time, however, a larger committee aggregates dispersed information more effectively. Holding fixed the candidates' choices, increasing the number of committee members makes it more likely that the candidate with the stronger underlying performance is selected; this is the Condorcet Jury Theorem at play. In most settings, this second force, which we refer to as the uncertainty effect, raises the value of investment by making the selection process more responsive to differences between candidates. Committee size therefore creates a trade-off between two opposing effects: a larger committee improves informational aggregation, but it also makes pivotal events rarer.

The analysis shows that these two forces generate a non-monotonic relationship between committee size and aggregate investment. In asymmetric environments (those where candidates are not equal), small increases in committee size can strengthen incentives because they improve the link between candidates' investments and the final outcome. Beyond a point, however, the decline in the likelihood of pivotality dominates, and further increases in committee size reduce aggregate investment. As a result, the committee size that maximizes candidates' investment is interior. This is in stark contrast with symmetric environments, where the informational benefit of adding committee members dominates throughout, so aggregate investment increases with committee size until it reaches its maximal level.

This paper is related to the political economy literature on voting, committees, and collective decision-making. Palfrey and Rosenthal (1983) developed the classic pivotal-voter logic: an individual's action affects outcomes only when that individual is decisive. Austen-Smith and Banks (1996) extend this perspective to committees with private information, showing how pivotal reasoning shapes sincere voting and information aggregation. Our paper applies this logic to the incentives of candidates rather than only to the behaviour of voters or committee members.

One of the closest papers to ours is Snyder (1989), who studies a two-candidate environment and compares settings in which the objective is to win the most battles with settings in which the objective is to win a majority of battles. Our contribution

with respect to Snyder (1989) is different in both emphasis and substance. In Snyder (1989), the fact that strategic incentives depend on the probability that victories are evenly split across sides is noted but not developed. By contrast, that knife-edge event is the central object in our analysis. We also go beyond Snyder (1989) in two further respects. First, we allow for parameter values under which payoffs are not globally concave, and find that this has qualitative implications for equilibrium behaviour. Second, unlike Snyder (1989), we provide tight conditions for the existence of pure-strategy equilibria.

Klumpp and Polborn (2006) are also closely related, as they connect majority-based competition to political selection in the context of U.S. primaries. The key difference is that their main analysis focuses on symmetric candidates and symmetric equilibria, whereas we allow for asymmetries between candidates. In our analysis, that symmetric case is precisely the exceptional benchmark in which the pivotality mechanism is muted: because each side is equally likely to receive support from any given committee member, the probability of a close vote declines much more slowly relative to the gains from improved aggregation. Once even small asymmetries are introduced, however, committee size creates a trade-off between information aggregation and pivotality. It is this trade-off that generates our main comparative-static result: aggregate investment is generally non-monotonic in committee size, and the investment-maximizing committee size is interior.

The paper is also related to the broader literature on contest theory. The classic foundations are Tullock (1980), Hirshleifer (1989), Skaperdas (1996), and Clark and Riis (1998). Moldovanu and Sela (2001) and Konrad (2009) provide broader treatments of prize allocation and strategic behaviour, while Corchón and Serena (2018) offer a recent overview of the literature. At a more technical level, Corchón and Dahm (2010) study the foundations of contest success functions, and Rai and Sarin (2009) analyse generalized specifications.

Within this broader literature, the paper is also connected to work on equilibrium structure, asymmetry, and comparative statics in two-candidate settings. Baye et al.

(1996) characterize the all-pay auction benchmark, which provides an important reference point when pure strategy equilibria fail to exist. Linster (1993) studies Stackelberg behaviour, Wang (2010) examines optimal accuracy in asymmetric contests, Feng and Lu (2017) derive uniqueness results for asymmetric Tullock environments, and Ewerhart (2015), Ewerhart (2017A), and Ewerhart (2017B) develop the theory of mixed equilibria, small noise, and revenue ranking in related models. Alcalde and Dahm (2010) provide neutrality results on rent dissipation. These papers are important for our analysis because they help characterize when equilibria are in pure strategies, semi-mixed strategies, or all-pay-type mixed strategies. Our contribution relative to this work is, on top of using a new success function, a new mechanism generated by collective decision-making: the pivotality effect.

The paper is further related to the literature on simultaneous multi-battle competition and other settings in which success depends on combinations of intermediate outcomes. Szentes and Rosenthal (2003a, 2003b) study simultaneous auctions in which a majority of objects matters, Kovenock and Roberson (2010) survey conflicts with multiple battlefields, and Arbatskaya and Mialon (2010) analyse multi-activity contests. Stephenson (2024) studies multi-battle environments with complementary battlefields, while Deku et al. (2015) examine strategic complementarities in simultaneous multi-battle settings. Fu, Lu, and Pan (2015) study team contests with multiple pairwise battles, and Feng et al. (2024) analyse optimal prize design in team contests with pairwise battles. We focus instead on a single investment directed at one committee decision, which allows us to isolate how majority rule itself changes incentives.

2 The Model

Consider a setting in which two candidates compete for a position worth $v > 0$. To improve their chances of being selected, each candidate $i \in \{1, 2\}$ simultaneously chooses an investment level, denoted by $x_i \geq 0$. Investment is costly, with investment

cost function

$$c_i(x_i) = \alpha_i x_i,$$

where $\alpha_i > 0$ for each $i \in \{1, 2\}$. After both candidates choose their investment levels, a committee determines which candidate is selected.

The committee has $N \geq 1$ members, where N is odd. Each committee member simultaneously supports one of the two candidates, and the position is awarded to the candidate who receives support from the majority of the committee.

Conditional on the candidates' investment levels, committee members' support decisions are independent and identically distributed. The probability that a given committee member supports candidate i is denoted by p_i and is given by the Tullock form

$$p_i(x_i, x_j) = \begin{cases} \frac{n_i x_i^m}{n_i x_i^m + n_j x_j^m} & \text{if } x_i + x_j \neq 0, \\ \frac{1}{2} & \text{otherwise,} \end{cases}$$

where $n_i > 0$ and $m > 0$ for all $i \in \{1, 2\}$ and $j \in \{1, 2\} \setminus i$. Whenever convenient, we suppress the investment arguments of p_i .

This completes the description of the model. In Section 4 we consider a number of extensions and alternative assumptions.

3 Results

3.1 Equilibrium

We break down the equilibrium analysis into three parts. In the first part, we study pure strategy equilibria. In the second part, we look at semi-mixed equilibria, where one candidate uses a deterministic investment level and the other mixes between a positive investment level and zero investment. Finally, we consider a fully mixed strategy equilibrium.

Pure-strategy equilibrium

The payoff of candidate i as a function of investment levels is given by π_i , where

$$\pi_i = v \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} p_i^k (1-p_i)^{N-k} - c_i(x_i).$$

At the core of the pivotality effect is an equality that relates the Binomial cumulative distribution function with the Incomplete Beta Function:

$$\sum_{k=\frac{N+1}{2}}^N \binom{N}{k} p_i^k (1-p_i)^{N-k} = \frac{N!}{\frac{N-1}{2}!2} \int_0^{p_i} y^{\frac{N-1}{2}} (1-y)^{\frac{N-1}{2}} dy. \quad (1)$$

This result is featured in Wadsworth (1960) and Abramowitz and Stegun (1965), among others. For completeness, we present the proof for a general voting rule q in Appendix A1 (simple majority is given by $q = \frac{1}{2}$). The proof involves iteratively integrating by parts on the right-hand side of the equation above.

The convenience of (1) is that it allows us to easily derive the best responses of candidates without having to deal with a sum where the number of terms depends on the committee size. The best response of candidate i , given the investment level of the other candidate, is found by setting $\frac{\partial \pi_i}{\partial x_i} = 0$ and is implicitly given by

$$v \frac{N!}{\frac{N-1}{2}!2} p_i^{\frac{N-1}{2}} (1-p_i)^{\frac{N-1}{2}} \frac{\partial p_i}{\partial x_i} = c'_i(x_i). \quad (2)$$

With only one committee member, we have that $v \frac{\partial p_i}{\partial x_i} = c'_i(x_i)$. Thus, the addition of additional committee members introduces the term $\frac{N!}{\frac{N-1}{2}!2} p_i^{\frac{N-1}{2}} (1-p_i)^{\frac{N-1}{2}}$, which is equal to the probability that the committee is exactly one vote away from selecting either candidate. That is, $\frac{N-1}{2}$ committee members are choosing candidate i and $\frac{N-1}{2}$ committee members are choosing candidate j , while the remaining committee member is still undecided. Using the terminology from the voting literature, $\frac{N!}{\frac{N-1}{2}!2} p_i^{\frac{N-1}{2}} (1-p_i)^{\frac{N-1}{2}}$ is the probability that there is one pivotal committee member. This is the pivotality effect.

Given equation (2), we have that the best response investment levels are related via the equation

$$\frac{c'_1(x_1)}{c'_2(x_2)} = \frac{\frac{\partial p_1}{\partial x_1}}{\frac{\partial p_2}{\partial x_2}}. \quad (3)$$

Note that in equation (3), the committee size N is not present. While N affects the level of investment chosen by both candidates, it does not affect how the best responses are related.

Substituting the functions c_i and p_i , we obtain that the best response investment levels are related via the equation

$$\frac{x_2}{x_1} = \frac{\alpha_1}{\alpha_2}. \quad (4)$$

Define $\theta = \frac{n_2}{n_1} \left(\frac{\alpha_1}{\alpha_2} \right)^m$. The equation above, together with equation (2), leads to

$$\begin{aligned} \alpha_1 &= v \frac{N!}{\frac{N-1}{2}!^2} \left(\frac{x_1^m}{x_1^m + \theta x_1^m} \right)^{\frac{N-1}{2}} \left(\frac{\theta x_1^m}{x_1^m + \theta x_1^m} \right)^{\frac{N-1}{2}} \\ &\quad \times \frac{m x_1^m \theta x_1^m}{(x_1^m + \theta x_1^m)^2} \frac{1}{x_1}, \\ \alpha_1 x_1 &= v \frac{N!}{\frac{N-1}{2}!^2} \left(\frac{\theta}{(1+\theta)^2} \right)^{\frac{N-1}{2}} \frac{m\theta}{(1+\theta)^2}, \\ x_1 &= v \frac{N!}{\frac{N-1}{2}!^2} \left(\frac{\theta}{(1+\theta)^2} \right)^{\frac{N+1}{2}} \frac{m}{\alpha_1}, \end{aligned}$$

Since $x_2 = \frac{\alpha_1}{\alpha_2} x_1$, we have that

$$x_i = v \frac{N!}{\frac{N-1}{2}!^2} \left(\frac{\theta}{(1+\theta)^2} \right)^{\frac{N+1}{2}} \frac{m}{\alpha_i} \quad (5)$$

for $i \in \{1, 2\}$.

Note that if an equilibrium in pure strategies exists, it is unique. This is because we have just proven by construction that there is a unique value for the pair (x_1, x_2) at which both candidates play a best response.

Finally, for the pair of investment levels in (5) to be an equilibrium, it must be that candidates' profits are positive. This is a necessary and sufficient condition: using second-order conditions (see Appendix A2), for any given investment level of candidate $-i$, the profit function of candidate i is either concave, or convex for some investment level $x_i \in [0, \hat{x}]$ and concave for $x_i \in [\hat{x}, +\infty)$. Therefore, since $\pi_i(0, x_{-i}) = 0$, a critical point at which profits are positive is necessarily a global maximizer.

Define $p = \frac{1}{1+\theta}$. The profits of both candidates are positive in equilibrium if and only if

$$\min \left\{ \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} p^k (1-p)^{N-k}, \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} (1-p)^k p^{N-k} \right\} - \frac{N!}{\frac{N-1}{2}!^2} [p(1-p)]^{\frac{N+1}{2}} m \geq 0.$$

Define $f(N, m, \theta)$ as the left-hand side in the inequality above. We have that if $f(N, m, \theta) \geq 0$ then the profits of both candidates at the investment levels in (5) are positive and, thus, we have a unique pure strategy equilibrium. We have just proven the following:

Proposition 1. *If $f(N, m, \theta) \geq 0$ there exists a unique pure strategy equilibrium where investment levels are given by (5).*

There are cases in which the relation f takes a simple enough form to use it analytically, such as in the next result.

Corollary 1. *In a fully symmetric setting ($n_1 = n_2$ and $\alpha_1 = \alpha_2$), an equilibrium in pure strategies exists, and is unique, if and only if $\frac{N!}{\frac{N-1}{2}!^2} \frac{1}{2^N} m \leq 1$.*

In the case with only one committee member ($N = 1$) and a symmetric setting ($\alpha_1 = \alpha_2$ and $n_1 = n_2$), we have that a pure strategy equilibrium exists, and it is unique, if and only if $m \leq 2$. This is a well-known result in the contest literature (see Tullock (1980) and Baye et al. (1996)).

If candidates are symmetric ($\alpha_1 = \alpha_2$) and treated symmetrically by the committee members ($n_1 = n_2$), as in the case considered in Klumpp and Polborn (2006), we obtain the same necessary and sufficient existence conditions as they do (Corollary 1).

Snyder (1989) shows that a sufficient condition for a unique pure strategy equilibrium for a setting similar to ours is $mN \leq 1$, and acknowledges that this condition is not tight. Our function f readily allows for a tighter simple condition for all n_1, n_2, α_1 and α_2 : $m \frac{N+1}{2} \leq 1$, and also the necessary and sufficient conditions for the case where $m \frac{N+1}{2} > 1$ (in this case, the conditions depend on the particular values of n_1, n_2, α_1 and α_2 , as given by f implicitly).

If $m \frac{N+1}{2} > 1$, the profit function is not globally concave. Pure strategy equilibria where the profit function is not globally concave are possible. To our knowledge, the case with $N > 1$ where the profit function is not globally concave ($m \frac{N+1}{2} > 1$) has not been looked at before in the literature.

A situation where the profit function is not globally concave is depicted in Figure 1. As can be seen, in a neighbourhood around zero investment, the shape of the profit function is fundamentally different between $N = 1$ and $N = 3$. With a single committee member ($N = 1$), the profit function is globally concave, as starting from zero investment, a marginal increase in investment increases the candidate's probability of winning by a proportionately higher amount compared to the increase in investment. On the other hand, with a committee of more than one member, the profit function is convex near 0. This is because a marginal increase in investment of $\Delta > 0$ increases the probability of winning by a factor of Δ^2 or lower, which is negligible for small enough Δ . That is, at an investment cost of order Δ and a benefit of order Δ^2 , the candidate goes from zero profit to negative profit. As we shall see later on, this effect has qualitative implications, such as fostering the crowding out of candidates.

Semi-Mixed Strategy Equilibrium

We now turn to study the semi-mixed strategy equilibrium. We refer to a semi-mixed strategy equilibrium as one where a candidate, say i , chooses a deterministic

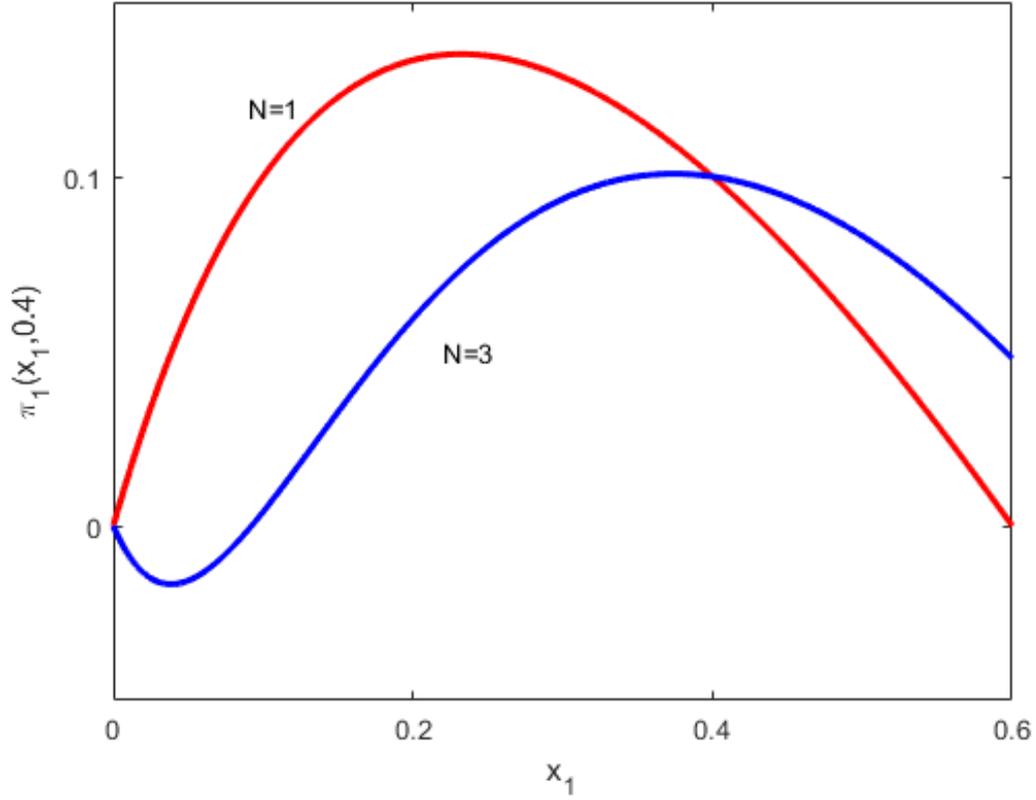


Figure 1: Example of the profit function of candidate 1 when candidate 2 chooses investment $x_2 = 0.4$ with $v = \alpha_1 = \alpha_2 = n_1 = n_2 = m = 1$.

level of investment x_i , whereas the other candidate mixes between two deterministic levels, a positive one x_{-i} , with probability $\delta \in [0, 1]$, and zero 0 with probability $1 - \delta$. We consider such equilibria in this section.

Since a candidate is mixing a positive investment level with a zero investment level, their profits when choosing the positive level of investment must be zero. Using this fact together with the analysis in our previous section, we can obtain the equations

needed to find the equilibrium values of x_i , x_{-i} and δ . We have:

$$\delta v \frac{N!}{\frac{N-1}{2}!^2} (p_i(1-p_i))^{\frac{N+1}{2}} \frac{m}{x_i} - \alpha_i = 0, \quad (6)$$

$$v \frac{N!}{\frac{N-1}{2}!^2} (p_i(1-p_i))^{\frac{N+1}{2}} \frac{m}{x_{-i}} - \alpha_{-i} = 0, \quad (7)$$

$$v \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} (1-p_i)^k p_i^{N-k} - \alpha_{-i} x_{-i} = 0, \quad (8)$$

where the first two equations are the first-order conditions of candidates i and $-i$ respectively, and the third one is the zero profit condition for candidate $-i$ when they choose investment x_{-i} .

Notice that from (6) and (7) we have that $\frac{x_i}{x_{-i}} = \frac{\delta \alpha_{-i}}{\alpha_i}$. This means that $p_i = \hat{p} = \frac{1}{1+\hat{\theta}}$ with $\hat{\theta} = \frac{n_2}{n_1} \left(\frac{\alpha_1}{\delta \alpha_2} \right)^m$. Plugging (7) into (8) with this fact leads to

$$\sum_{k=\frac{N+1}{2}}^N \binom{N}{k} (1-\hat{p})^k \hat{p}^{N-k} - \frac{N!}{\frac{N-1}{2}!^2} (\hat{p}(1-\hat{p}))^{\frac{N+1}{2}} \frac{m}{\alpha_{-i}} = 0,$$

where the only unknown is δ (through $\hat{p} = \frac{1}{1+\hat{\theta}}$ with $\hat{\theta} = \frac{n_2}{n_1} \left(\frac{\alpha_1}{\delta \alpha_2} \right)^m$). Solving this equation leads to an equilibrium value for δ . Plugging this value into (7) gives an equilibrium value for x_{-i} . Finally, using the relation $\frac{x_i}{x_{-i}} = \frac{\delta \alpha_2}{\alpha_1}$ with the values for δ and x_{-i} , gives an equilibrium value for x_i .

From Ewerhart (2017A), Alcalde and Dahm (2010) and Wang (2010), we know that if $\frac{N!}{\frac{N-1}{2}!^2} \left(\frac{1}{2} \right)^N m > 1$ then the equilibrium in our model is in mixed strategies (not just semi-mixed), and has features shared by the all-pay auction equilibrium (Baye et al. (1996)). Therefore, the semi-mixed strategy equilibrium exists for parameter constellations such that a pure strategy equilibrium does not exist (as it is unique), i.e. $f(N, m, \theta) < 0$, and $\frac{N!}{\frac{N-1}{2}!^2} \left(\frac{1}{2} \right)^N m \leq 1$. Moreover, from Lemma 5 in Appendix A3 we have that whenever $\frac{N!}{\frac{N-1}{2}!^2} \left(\frac{1}{2} \right)^N m \leq 1$, the equilibrium in our game is unique. We collect these findings in our next result.

Proposition 2. *Assume that $f(N, m, \theta) < 0$ and $\frac{N!}{\frac{N-1}{2}!^2} \left(\frac{1}{2} \right)^N m \leq 1$. There exists a unique semi-mixed strategy equilibrium given by (6)-(8).*

All-Pay Equilibrium

Ewerhart (2017A) shows that if $\frac{N!}{\frac{N-1}{2}!2} \frac{1}{2^N} m > 1$ then the all-pay equilibrium with its characteristics ensues in settings such as ours. We rephrase this for our setting:

Proposition 3. *Assume without loss of generality that $\frac{n_2}{n_1} \left(\frac{\alpha_1}{\alpha_2}\right)^m \leq 1$. If*

$$\frac{N!}{\frac{N-1}{2}!2} \frac{1}{2^N} m > 1,$$

then in all equilibria the expected investment of candidate 1 is $\frac{v}{2\alpha_2}$, and the expected investment of candidate 2 is $\frac{v}{2\alpha_2} \frac{\alpha_1}{\alpha_2}$.

A full characterization of the mixed strategies used by candidates in the all-pay equilibrium is presented in Ewerhart (2015). There it is shown that the mixed strategy equilibrium has candidates choose investment levels that belong to a countably infinite set of mass points, with zero as the only accumulation point of the investment levels. The particular structure of the equilibrium is not relevant for the purpose of this paper, only the value of average investment levels is.

Equilibrium Characterization

We collect the analysis above in the following theorem.

Theorem 1. *Define $p = \frac{1}{1+\theta}$ with $\theta = \frac{n_2}{n_1} \left(\frac{\alpha_1}{\alpha_2}\right)^m$. Assume without loss of generality that $\theta \leq 1$.*

1. *If*

$$\sum_{k=\frac{N+1}{2}}^N \binom{N}{k} (1-p)^k p^{N-k} - \frac{N!}{\frac{N-1}{2}!2} [p(1-p)]^{\frac{N+1}{2}} m \geq 0,$$

there exists a unique pure strategy equilibrium where investment levels are given by

$$x_i = v \frac{N!}{\frac{N-1}{2}!2} \left(\frac{\theta}{(1+\theta)^2} \right)^{\frac{N+1}{2}} \frac{m}{\alpha_i},$$

for $i \in \{1, 2\}$.

2. If

$$\sum_{k=\frac{N+1}{2}}^N \binom{N}{k} (1-p)^k p^{N-k} - \frac{N!}{\frac{N-1}{2}!^2} [p(1-p)]^{\frac{N+1}{2}} m < 0,$$

and $\frac{N!}{\frac{N-1}{2}!^2} \frac{1}{2^N} m \leq 1$, there exists a unique semi-mixed strategy equilibrium where candidate 1 chooses investment level x_1 , and candidate 2 chooses investment level x_2 with probability $\delta \in [0, 1]$ and investment level 0 with probability $1 - \delta$, where x_1, x_2 and δ are found by solving:

$$\begin{aligned} \delta v \frac{N!}{\frac{N-1}{2}!^2} (\hat{p}(1-\hat{p}))^{\frac{N+1}{2}} \frac{m}{x_1} - \alpha_1 &= 0, \\ v \frac{N!}{\frac{N-1}{2}!^2} (\hat{p}(1-\hat{p}))^{\frac{N+1}{2}} \frac{m}{x_2} - \alpha_2 &= 0, \\ v \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} (1-\hat{p})^k \hat{p}^{N-k} - \alpha_2 x_2 &= 0, \end{aligned}$$

where $\hat{p} = \frac{1}{1+\hat{\theta}}$ with $\hat{\theta} = \frac{n_2}{n_1} \left(\frac{\alpha_1}{\delta \alpha_2} \right)^m$.

3. If $\frac{N!}{\frac{N-1}{2}!^2} \frac{1}{2^N} m > 1$, then all equilibria are in mixed strategies and such that the expected investment of candidate 1 is $\frac{v}{2\alpha_2}$ and the expected investment of candidate 2 is $\frac{v}{2\alpha_2} \frac{\alpha_1}{\alpha_2}$.

Figure 2 shows the regions where each type of equilibrium applies as a function of the committee size N and power m , for given values of the parameters α_1, α_2, n_1 and n_2 .

3.2 Aggregate Investment

Next we move to the analysis of aggregate investment as a function of the committee size, where aggregate investment is defined as the sum of expected investment levels in equilibrium, $E(x_1) + E(x_2)$. We start the analysis with the case where candidates are symmetric, $\theta = \frac{n_2}{n_1} \left(\frac{\alpha_1}{\alpha_2} \right)^m = 1$, as it is qualitatively different than the asymmetric case.

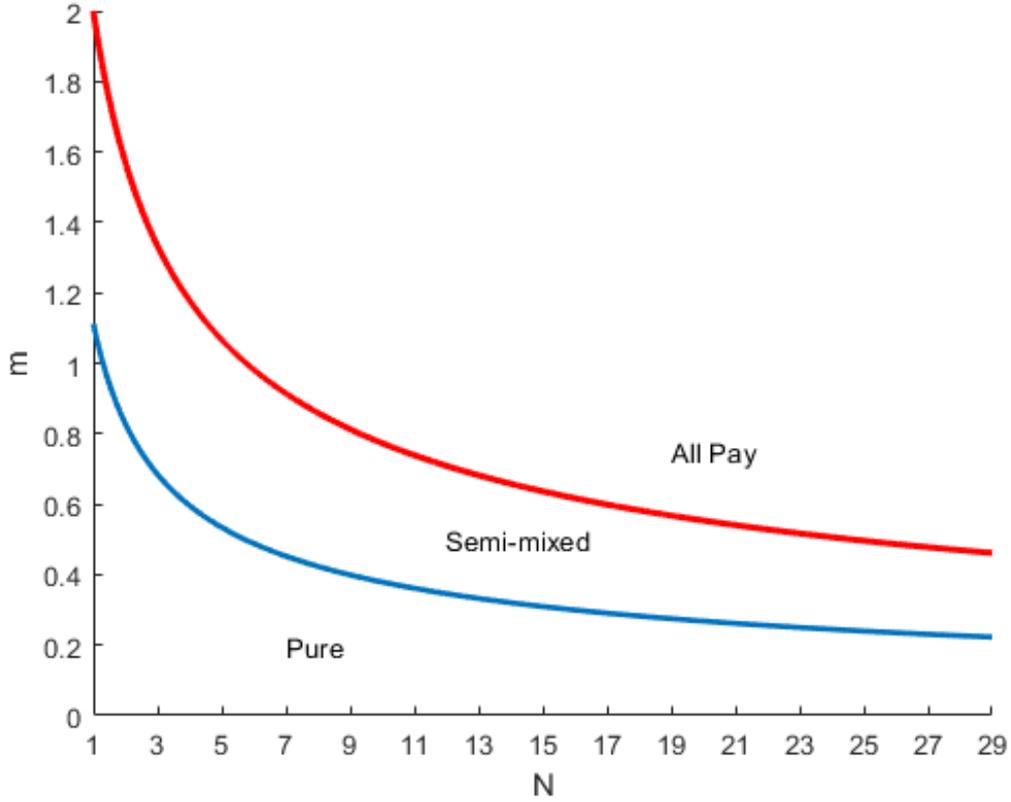


Figure 2: Regions for the different types of equilibria when $\alpha_1 = 0.14$ and $v = \alpha_2 = n_1 = n_2 = 1$.

In the symmetric case, Corollary 1 states that an equilibrium in pure strategies exists if and only if $m \leq \hat{m} = 2^N \frac{N-1!^2}{N!}$, in which case the sum of equilibrium investments is $\frac{v}{\alpha} \frac{m}{\hat{m}(N)}$ by Proposition 1. Moreover, as shown by Theorem 1, if $m > \hat{m}(N)$ the sum of expected equilibrium investments is given by $\frac{v}{\alpha}$. This leads to our next result.

Proposition 4. *Assume $n_1 = n_2$ and $\alpha_1 = \alpha_2 = \alpha$, and define*

$$\hat{m}(N) = 2^N \frac{N-1!^2}{N!}.$$

If $m \leq \hat{m}(N)$ the sum of equilibrium investments equals $\frac{v}{\alpha} \frac{m}{\hat{m}(N)}$, whereas if $m > \hat{m}(N)$ the sum of expected equilibrium investments is $\frac{v}{\alpha}$.

Figure 3 shows an example of the sum of expected investments for different committee sizes. Note that the plot should consist of discrete points instead of a smooth curve, as the number of committee members is a discrete number. However, here and henceforth we use the Regularized Beta Function to plot continuous curves, as we believe this helps to visualize trends and compare different scenarios more easily.

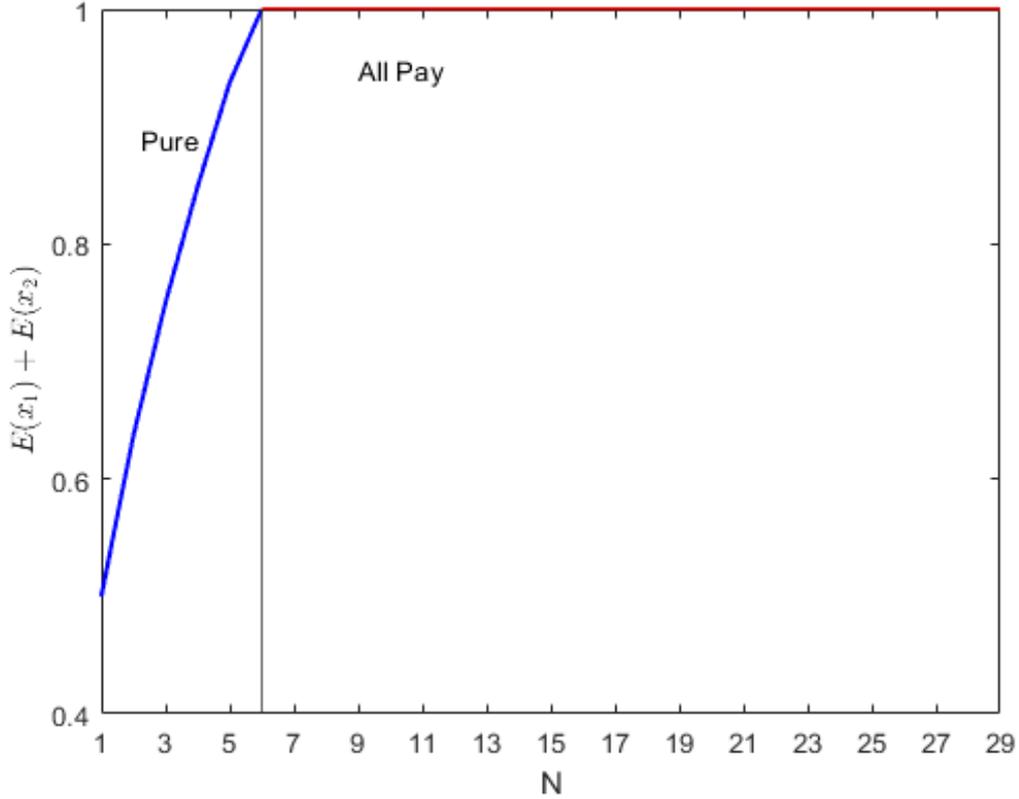


Figure 3: Sum of equilibrium investments when $v = \alpha_1 = \alpha_2 = n_1 = n_2 = m = 1$.

Notice how the sum of expected investments is increasing with committee size, and maximised for any N such that a pure strategy equilibrium is no longer possible. As shown in Proposition 4, this observation holds for symmetric settings. However, as we shall see next, this is not the case for asymmetric settings, where the sum of investments is non-monotonic in committee size because of the pivotality and uncertainty effects.

Consider now the case where candidates are asymmetric ($\theta \neq 1$). First, we explore how committee size affects the investments of both candidates in the pure strategy equilibrium. Note that, since in the pure strategy equilibrium $x_2 = \frac{\alpha_1}{\alpha_2}x_1$, we have that maximising $x_1 + x_2$ is equivalent to maximising x_1 .

From (5), maximising x_1 is equivalent to maximising $A(N)$ given by

$$A(N) = \frac{N!}{\frac{N-1}{2}!2} \left(\frac{\theta}{(1+\theta)^2} \right)^{\frac{N+1}{2}}.$$

Define $D(N) = A(N+2) - A(N)$. If $D(N) > 0$, then both candidates' equilibrium investments increase when the committee size goes from N to $N+2$. We have,

$$D(N) \propto \frac{N+2}{N+1} \frac{4\theta}{(1+\theta)^2} - 1. \quad (9)$$

Note that $D(N)$ is decreasing in N . Moreover, $\frac{N+2}{N+1} > 1$ for all N , whereas

$$\frac{4\theta}{(1+\theta)^2} \leq 1$$

with equality only when $\theta = 1$. Therefore, ignoring equilibrium payoffs for now, if $\theta \neq 1$, then there exists a finite $N \geq 1$ that maximises equilibrium investment. The investment-maximising committee size is either the odd floor or the odd ceiling of the N^* that solves $D(N^*) = 0$. We have the following result:

Proposition 5. *Assume asymmetric candidates, and let \hat{N} be the committee size that solves $D(\hat{N}) = 0$. If*

$$\sum_{k=\frac{\hat{N}+1}{2}}^{\hat{N}} \binom{\hat{N}}{k} (1-p)^k p^{\hat{N}-k} - \frac{\hat{N}!}{\frac{\hat{N}-1}{2}!2} [p(1-p)]^{\frac{\hat{N}+1}{2}} m \leq 0,$$

with $p = \frac{1}{1+\theta}$ and $\theta = \frac{n_2}{n_1} \left(\frac{\alpha_1}{\alpha_2} \right)^m$, then either the odd-floor or the odd-ceiling of \hat{N} maximizes the expected sum of equilibrium investments.

The reason why there can be an optimal interior investment-maximising committee size is that there are two incentives at play. On the one hand, increasing the

committee size reduces uncertainty, i.e. the candidate with a higher investment is more likely to win, which in settings with not too heterogeneous candidates leads to an increase in investment; this is the uncertainty effect. On the other hand, increasing the committee size also leads to a reduction in the probability that a committee member is pivotal and, thus, a reduction in investment; this is the pivotality effect. With low committee size, the uncertainty effect dominates, and increasing the number of committee members leads to more investment. However, as committee size increases, there comes a point where the pivotal effect starts to dominate, and increasing the number of committee members leads to a decrease in equilibrium investment.

The model considered in Klumpp and Polborn (2006) is such that $\alpha_1 = \alpha_2 = 1$, and $n_1 = n_2 = 1$. The situation in Klumpp and Polborn (2006) is the symmetric case: candidates' investment costs are symmetric, and committee members treat candidates symmetrically. With their restrictions, we have $\theta = 1$, which means that $\frac{4\theta}{(1+\theta)^2} = 1$ for all $m > 0$, and thus $D(N)$ is always positive for all N . This is a special case that does not appear as soon as either candidates are not symmetric or committee members do not treat candidates symmetrically. In this paper, we drop all these symmetry assumptions (note that Klumpp and Polborn (2006) consider asymmetric candidates, but only for the limit case of $N \rightarrow \infty$).

The intuition for why the pivotality effect is not relevant in fully symmetric settings is that $\frac{4\theta}{(1+\theta)^2} = 1$ implies that in equilibrium $p_1 = p_2 = \frac{1}{2}$. In a symmetric setting, and only then, the probability that a committee member is pivotal vanishes at rates orders of magnitude lower compared to the uncertainty effect. In particular, we have for all $N \geq 1$, including in the limit, that $\frac{N+2}{N+1} \frac{4\theta}{(1+\theta)^2} > 1$. That is, the pivotality effect never dominates the uncertainty effect. In this case, the investment maximising committee sizes are all N such that $m > \hat{m}(N)$, as discussed in the previous section.

Moving on to study aggregate investment in the semi-mixed strategy equilibrium, we have the following result whose proof is presented in Appendix A3.

Proposition 6. *In the semi-mixed strategy equilibrium, the expected sum of invest-*

ments decreases as N increases, and for N large enough it converges to $\frac{v}{2\alpha_2} \left(1 + \frac{\alpha_1}{\alpha_2}\right)$.

Therefore, from Proposition 6 we can deduce that the investment maximizing committee size is not one that leads to a semi-mixed strategy.

To conclude, from Proposition 3, we have the following result with respect to the sum of investments when the all-pay equilibrium ensues.

Proposition 7. *For any $N' > N$ where $\frac{N!}{\frac{N-1}{2}!^2} \left(\frac{1}{2}\right)^N m \geq 1$, the expected sum of equilibrium investments equals $\frac{v}{2\alpha_2} \left(1 + \frac{\alpha_1}{\alpha_2}\right)$.*

Collating the three previous results, we have the following characterization for the equilibrium sum of investments.

Theorem 2. *With asymmetric candidates, there exists an N^* that maximizes the expected sum of equilibrium investments.*

Let \hat{N} be the committee size that solves $D(\hat{N}) = 0$. If

$$\sum_{k=\frac{\hat{N}+1}{2}}^{\hat{N}} \binom{\hat{N}}{k} (1-p)^k p^{\hat{N}-k} - \frac{\hat{N}!}{\frac{\hat{N}-1}{2}!^2} [p(1-p)]^{\frac{\hat{N}+1}{2}} m \leq 0,$$

with $p = \frac{1}{1+\theta}$ and $\theta = \frac{n_2}{n_1} \left(\frac{\alpha_1}{\alpha_2}\right)^m$, then N^* equals either the odd-floor or the odd-ceiling of \hat{N} . Otherwise, N^* equals either the odd-floor or the odd-ceiling of the N that solves $\sum_{k=\frac{N+1}{2}}^N \binom{N}{k} (1-p)^k p^{N-k} - \frac{N!}{\frac{N-1}{2}!^2} [p(1-p)]^{\frac{N+1}{2}} m = 0$

This result highlights the contrast between the symmetric and asymmetric cases. In the fully symmetric case, the uncertainty effect dominates throughout; increasing the number of committee members increases the probability of choosing the candidate that chose the highest investment without significantly weakening incentives to invest through pivotality, because in equilibrium each committee member is exactly indifferent between the two candidates. As a result, equilibrium investment is weakly increasing in committee size until the semi-mixed strategy equilibrium ensues, at which points equilibrium investment levels are constant on committee size.

By contrast, once even a small asymmetry is introduced, the pivotality effect becomes relevant. The stronger candidate is more likely to be selected by any given committee member (this is straightforward to see in the pure strategy equilibrium and is shown in Lemma 1 in Appendix A3 for the semi-mixed strategy equilibrium), so adding committee members reduces the probability that the committee is close enough for an individual vote to matter. This lowers the marginal return to investment and eventually outweighs the reduction in uncertainty generated by a larger committee. As a result, equilibrium investment is non-monotonic in committee size, and the investment-maximising committee size is interior. Figures 4 and 5 illustrate the fact that the relation between committee size and total investment is non-monotonic.

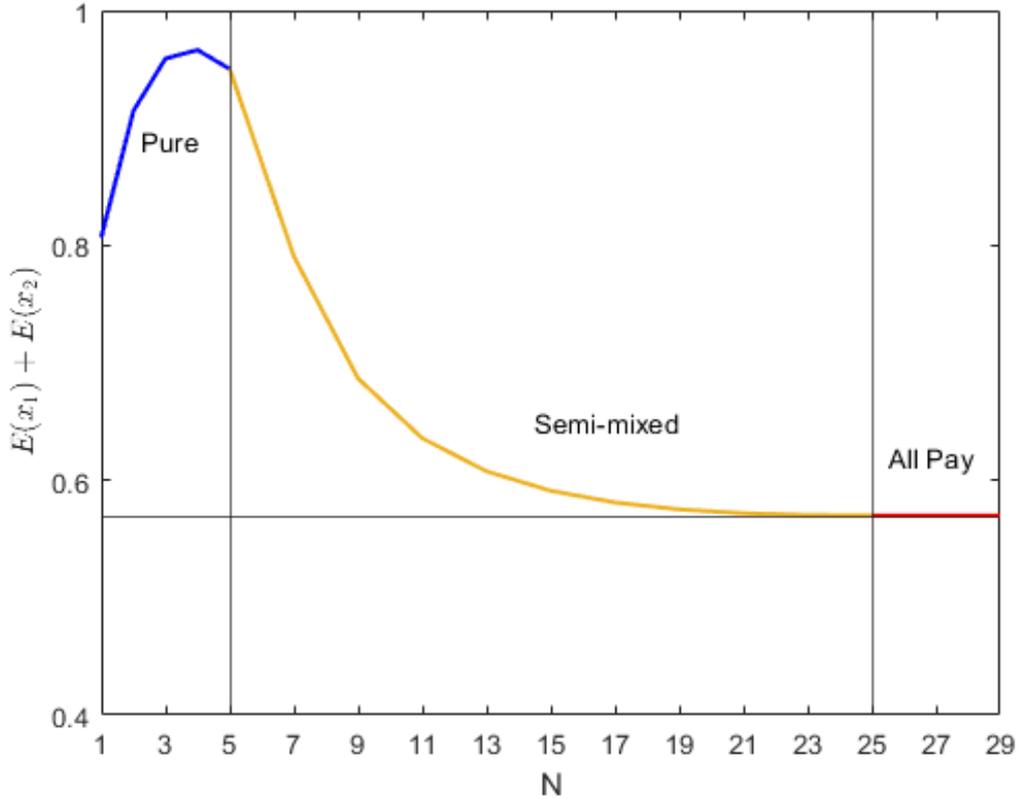


Figure 4: Sum of equilibrium investments when $\alpha_1 = 0.14$, $m = 0.5$, and $v = \alpha_2 = n_1 = n_2 = 1$. The sum of investments is maximized at $N^* = 3$.

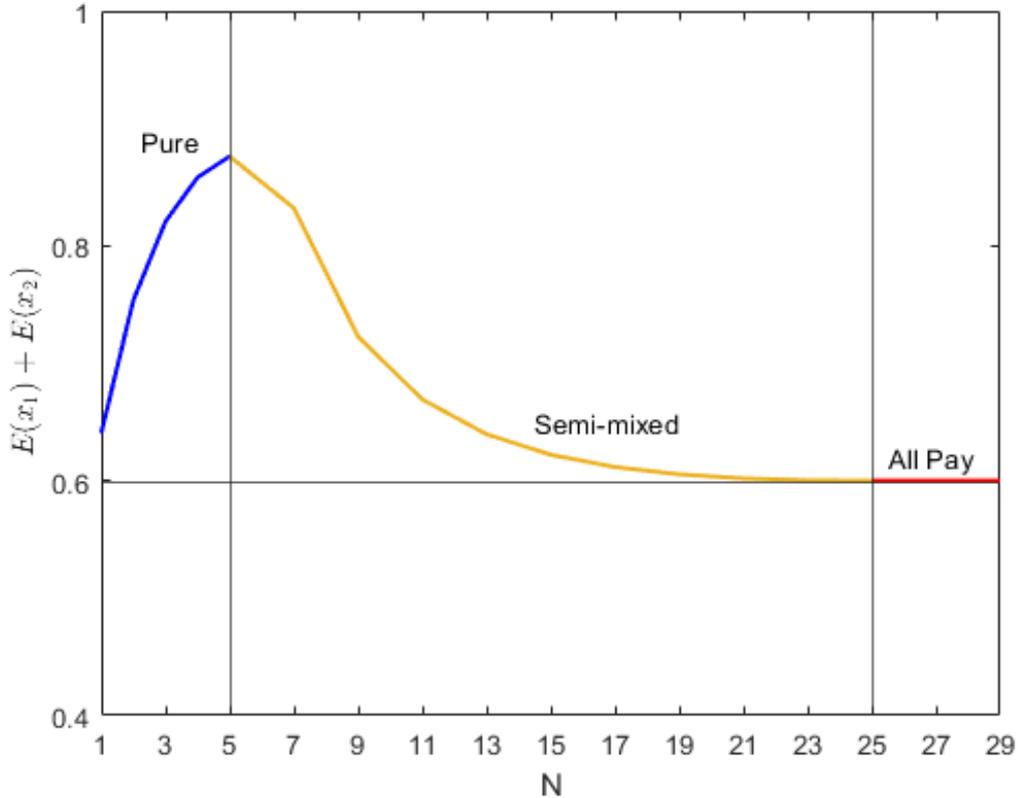


Figure 5: Sum of equilibrium investments when $\alpha_1 = 0.2$, $m = 0.5$, and $v = \alpha_2 = n_1 = n_2 = 1$. The sum of investments is maximized at $N^* = 5$.

4 Extensions

In this section we relax a number of the assumptions made here and show that the main qualitative results are robust. In Section 4.1, committee members are allowed to behave strategically rather than having their behaviour summarized by the reduced-form support function above. In Section 4.2, we consider supermajority rules and also allow for an even number of committee members. In Section 4.3, we study the case with more than two candidates.

It is useful to note that any environment with homogeneous valuations, heterogeneous marginal investment costs, and biased support probabilities, such as the one considered here, is strategically equivalent to an environment with heterogeneous val-

uations, homogeneous marginal investment costs, and an unbiased support function. Specifically, this equivalent formulation is obtained by setting $v_i = v \frac{n_i^{\frac{1}{m}}}{\alpha_i}$, and denoting candidate i 's investment in that formulation by \hat{x}_i , where $\hat{x}_i = n_i^{\frac{1}{m}} x_i$.

Note also that we assume that investment is one-dimensional. This differs from multi-battle formulations in which a potentially different investment level can be directed toward each separate arena of competition (or, in the present setting, toward each component decision). We focus on the one-dimensional case because it better captures the types of institutional settings we have in mind: a single selection decision made by a committee. The leading example is political selection, where candidates make a single broad investment in campaigning, visibility, and persuasion in order to secure support from a collective decision-making body. Other examples include grant competitions, in which each candidate submits one application that is evaluated by several reviewers, and judged performances such as gymnastics or diving, where a single performance is assessed by multiple evaluators.

4.1 Strategic Committee Members

We now explore in more detail committee members' behaviour by considering the case where committee members can be strategic. For this and the next sections we focus on pure strategy equilibria only.

A way to motivate committee members' strategic behaviour is by considering a situation where committee members want to choose the candidate that invested the most, i.e., they want candidate i to win if $x_i > x_j$ for $i \in \{1, 2\}$, and are indifferent about who wins if $x_1 = x_2$.

However, in many real-life scenarios, the investment of each candidate, or their type (i.e., their investment cost function) may be directly observable. Consider instead that each committee member k has a symmetric prior about candidates (i.e., their prior distribution about x_1 and α_1 is the same as that about x_2 and α_2), and, on top of that, each committee member receives an informative signal $\sigma_k \in \{1, 2\}$ about which candidate invested the most.

Consider, for instance, that signals are independent and identically distributed and are given by

$$P(\sigma_k = i | x_1, x_2) = p_i, \quad (10)$$

where p_i is as given in Section 2. Since committee members' prior is symmetric, if $\sigma_k = i$, then the probability that candidate i invested more than candidate j is higher than $\frac{1}{2}$, whereas if $\sigma_k = j$, then this probability is less than $\frac{1}{2}$.

Consider a situation where committee members want to choose the candidate that invested the most and use their signal σ_k from equation (10) to do so. In the main text, it was assumed that committee members' votes follow p_i , which with strategic committee members is equivalent to stating that they vote according to their signal. The question is then whether a situation where each committee member chooses a candidate according to their signal is a (Bayesian Nash) equilibrium. This is referred to in the voting literature as an informative equilibrium (see, for instance, Austen-Smith and Banks (1996)). The answer is positive.

Proposition 8. *If committee members are strategic, have a symmetric common prior, and receive an informative i.i.d. signal about investment choices given in (10), then there is an informative equilibrium where every committee member votes informatively.*

The proof follows from Austen-Smith and Banks (1996) and is thus omitted. The intuition is that a committee member must behave as if pivotal (i.e., the vote of the other committee members is split 50/50) because otherwise, their vote is irrelevant. If a committee member is pivotal and if committee members vote according to their signal, then there are as many signals for candidate 1 as there are for candidate 2 when not counting a committee member's own signal. Since the prior is symmetric and signals are i.i.d., the posterior when considering being pivotal and after observing a committee member's own signal is simply the committee member's own signal. Given this posterior, it is a best response to vote according to the signal as the signal has a more than 50% chance of being correct.

Note that, just as in Austen-Smith and Banks (1996), if we used a non-majority voting rule, then there are equilibria where some committee members are not sincere to offset whichever bias the voting rule might introduce. For example, if there are 7 committee members and candidate 1 wins if and only if this candidate is chosen by at least 6 committee members, there is no equilibrium where all committee members are sincere, but there is an equilibrium where any 4 committee members vote for candidate 1 and the other 3 vote sincerely. This setting is then equivalent to one where the voting rule is simple majority and there are only 3 committee members who are all sincere.

Since sincere voting is essentially what we assumed from the outset, the assumption can be rationalized if the appropriate informational setting and voting equilibrium is used.

4.2 Other Voting Rules and an Even Number of Committee Members

Suppose that N can take even values and candidate 1 wins if they are chosen by at least a fraction $q \in (0, 1]$ of committee members; otherwise, candidate 2 wins. We refer to q as the voting rule. For example, if $q = \frac{1}{2}$, then the voting rule is simple majority (this is the case we have looked at so far) where ties go to candidate 1 (alternatively, we could assume that ties imply that there is a draw, as in Vesperoni and Yildizparlak (2017)). On the other hand, if $q = 1$, the voting rule is unanimity for 1 with candidate 2 as the status quo.

The payoff of candidate 1 as a function of investment levels is given by

$$\pi_1 = v \sum_{k=\lceil Nq \rceil}^N \binom{N}{k} p_1^k (1-p_1)^{N-k} - \alpha_1 x_1,$$

and similarly for candidate 2.

Using the equivalence shown in Appendix A1, the critical points of π_i for $i \in \{1, 2\}$

are given by

$$v \frac{N!}{([\![Nq]\!] - 1)!(N - [\![Nq]\!])!} p_i^{[\![Nq]\!] - 1} (1 - p_i)^{N - [\![Nq]\!] } \frac{\partial p_i}{\partial x_i} = \alpha_i.$$

Best response investment levels are then related via the equation

$$\frac{\alpha_1}{\alpha_2} = \frac{x_2}{x_1}.$$

Note that this is the same equation as for simple majority, and that the voting rule q is not present. While N and q affect the equilibrium levels of investment, they do not affect how these investment levels are related.

Proceeding as in the proof of Proposition 1, a sufficient condition for an equilibrium in pure strategies is

$$m(\max\{[\![Nq]\!], N - [\![Nq]\!] \} + 1) \leq 1.$$

In terms of investment-maximising committee size, we proceed as in the main text. Increasing the committee size by 2 committee members (we choose 2 extra committee members instead of 1 to keep this section in line with the main model) increases the votes that i needs to win by $a = [q(N + 2)] - [qN] \in \{0, 1, 2\}$. Proceeding as in the case with $q = \frac{1}{2}$, and defining $D'(N)$ as the increase in equilibrium investment when going from N committee members to $N + 2$,

$$D'(N) \propto \frac{(N + 2)(N + 1)}{G(N, q)} 4p^a (1 - p)^{2-a} - 1.$$

where as in the main text $p = \frac{1}{1+\theta}$, and

$$G(N, q) = \begin{cases} 4(N - [\![Nq]\!] + 2)(N - [\![Nq]\!] + 1) & \text{if } a = 0, \\ 4[\![Nq]\!](N - [\![Nq]\!] + 1) & \text{if } a = 1, \\ 4([\![Nq]\!] + 1)[\![Nq]\!] & \text{if } a = 2, \end{cases}$$

with $a = [q(N + 2)] - [qN]$.

Note that the equation above is qualitatively the same as Equation (9), with the difference that increases in committee size have a varying effect on the sum of equilibrium investments depending on the rounding of natural numbers. Thus, the incentives and results about investment-maximising committee size are the same as those already presented.

4.3 More than Two Candidates

Assume for this section that there are $C \geq 3$ candidates and the winner is the candidate that is chosen by the most committee members, where for simplicity, in the case of a tie, a winner is chosen uniformly at random among tied candidates. Given investment levels, define a state of the world as a vector $\mathbf{v} \in \{1, \dots, N\}^C$ such that $\sum_i \mathbf{v}_i = N$. That is, \mathbf{v}_i denotes the votes received by candidate i .

If there are more than two candidates, deriving analytical results is no longer possible. The reason is that with more than two candidates, the number of votes needed to win depends on the whole distribution of votes that all candidates receive. As an example, if there are 3 candidates and 7 committee members, then candidate 1 wins in any of the following states of the world: (3, 2, 2), (4, 2, 1), (4, 1, 2), (5, 2, 0), (5, 0, 2), (5, 1, 1), (6, 1, 0), (6, 0, 1), (7, 0, 0), and (3, 3, 1) and (3, 1, 3) with probability $\frac{1}{2}$. As the number of candidates and/or the number of committee members increases, the number of states increases exponentially and, more importantly, there is no known analytically tractable way to enumerate them for general N and C .

Intuitively, however, our results carry through to the case with more than two candidates, as both the uncertainty effect and the pivotality effect are still present. Nevertheless, with three or more candidates, there is equilibrium multiplicity, and in particular, there can be equilibria where there is crowding out. Crowding out equilibria are those where a candidate, not necessarily the one with lower investment costs, chooses zero investment in equilibrium.

To illustrate these observations, we consider a setting with 3 candidates and committee sizes $N = 1$ and $N = 3$. Assume for simplicity that $v = 1$, $\alpha_1 = n_1 = m = 1$, $\alpha_2 = n_2 = 1$, $n_3 = 1$ and $\alpha_3 = \alpha > 0$. With these parameter values, we have that $p_i = \frac{x_i}{\sum_j x_j}$ for all $i \in \{1, 2, 3\}$, and $\alpha_i = 1$ for all $i \in \{1, 2\}$ and $\alpha_3 = \alpha$.

Let v_i denote the votes received by candidate i . The payoff of each candidate is

given by

$$\begin{aligned}\pi_1 &= P(\mathbf{v}_1 > \mathbf{v}_2, \mathbf{v}_1 > \mathbf{v}_3) + \frac{1}{3}P(\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}_3) - x_1, \\ \pi_2 &= P(\mathbf{v}_2 > \mathbf{v}_1, \mathbf{v}_2 > \mathbf{v}_3) + \frac{1}{3}P(\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}_3) - x_2, \\ \pi_3 &= P(\mathbf{v}_3 > \mathbf{v}_1, \mathbf{v}_3 > \mathbf{v}_2) + \frac{1}{3}P(\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}_3) - \alpha x_3.\end{aligned}$$

Note that with 3 candidates and committee sizes 1 and 3, there can never be a 2-way tie.

With a one-person committee ($N = 1$), it can be easily shown that for $\alpha \leq 2$, there is a unique pure strategy equilibrium given by $x_1 = x_2 = \frac{2\alpha}{(2+\alpha)^2}$ and $x_3 = \frac{2(2-\alpha)}{(2+\alpha)^2}$. If $\alpha \geq 2$, then there is an equilibrium where candidate 3 is crowded out and chooses no investment, whereas the other two candidates choose the same investment level (this is similar to the case in Franke (2012)). If $\alpha < 1$, i.e. candidate 3 has a lower investment cost function than those of candidates 1 and 2, then this candidate chooses more investment, whereas if $\alpha > 1$, then the opposite happens.

However, if the committee size is instead $N = 3$, we have that there is equilibrium multiplicity. To begin with, there is an equilibrium where all three candidates choose a positive investment level. On top of that, with $N = 3$, there are also equilibria with crowding out where one of the candidates chooses zero investment in equilibrium. In terms of crowding out candidates 1 or 2, if $0.646 < \alpha < 1.194$, there are two equilibria where $x_3 = \frac{6\alpha}{(1+\alpha)^4}$ and either $(x_1, x_2) = \left(\frac{6\alpha^2}{(1+\alpha)^4}, 0\right)$ or $(x_1, x_2) = \left(0, \frac{6\alpha^2}{(1+\alpha)^4}\right)$. A lower bound on α is needed because otherwise candidate 3's investment cost function is significantly lower than that of candidate 1, and this candidate would obtain negative profits in equilibrium. On the other hand, an upper bound on α is needed because otherwise candidate 3's investment cost function is significantly higher than that of candidates 1 and 2, and candidate 2 (or candidate 1) finds it profitable to enter the competition and choose a positive investment level.

In terms of crowding out candidate 3, for $\alpha > 0.906$, there is an equilibrium where candidates 1 and 2 choose $x_1 = x_2 = \frac{3}{8}$, and candidate 3 chooses $x_3 = 0$. Notice that there is crowding out of candidate 3 even in cases where this candidate has lower

investment costs than those of candidates 1 and 2. Such a situation is not possible with a single committee member.

The reason why there are equilibria with crowding out, even when the candidate crowded out is the one with the lowest investment cost function, is due to the pivotality effect. To see this, consider a situation where candidate 3 is not investing a positive amount and candidates 1 and 2 are investing positive amounts equal to the two-candidate equilibrium, i.e. the equilibrium investment levels in a situation where there is no candidate 3. Candidate 3's marginal profit at zero investment level is negative. The reason is that increasing investment marginally only improves payoff in cases where there is a pivotal committee member for candidate 3 (the pivotality effect), but since candidate 3 is currently investing zero, and the other two candidates are investing positive amounts, the probability of being in a pivotal state for candidate 3 is zero. Thus, a marginal increase in investment for candidate 3 will lead to negative profits.

Increasing the candidate count past 3 or using different parameter values leads to the same observations as just made for three candidates. We collect these in the following proposition, whose proof is via the example just shown.

Proposition 9. *With three or more candidates, there can be equilibrium multiplicity. Moreover, there can be crowding out equilibria where the candidate with the lowest investment cost function chooses zero investment.*

5 Conclusions

We considered committee-based selection when the winner is chosen by a committee. A committee introduces two distinct effects: an uncertainty effect and a pivotality effect. The pivotality effect occurs because marginal increases in investment increase marginal profit only when there is a pivotal committee member. As the committee size increases, the likelihood of a pivotal committee member decreases, which in turn reduces the returns to investment.

Our results have implications for committee design and candidates' aggregate investments. Future research could explore the pivotality effect both experimentally and empirically and, more broadly, how the presence of a committee affects incentives. Future work could also look at heterogeneous committee members and determine what the optimal committee composition is from the point of view of investment choice, or at strategic committee members where each committee member has a bias towards either candidate and receives information about investment choices that is not independent and identically distributed.

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Appendix

A1 - Proof of Equation (1)

We want to show the following equality for all $N \in \{1, 2, \dots\}$, $p \in [0, 1]$ and $q \in (0, 1]$:

$$\sum_{k=\lceil Nq \rceil}^N \binom{N}{k} p^k (1-p)^{N-k} = \frac{N!}{(\lceil Nq \rceil - 1)!(N - \lceil Nq \rceil)!} \int_0^p y^{\lceil Nq \rceil - 1} (1-y)^{N - \lceil Nq \rceil} dy.$$

Define $S = \sum_{k=\lceil Nq \rceil}^N \binom{N}{k} p^k (1-p)^{N-k}$. On the right-hand side of the equation above, set $u = (1-y)^{N - \lceil Nq \rceil}$ and $dv = y^{\lceil Nq \rceil - 1} dy$, then integrate by parts to obtain

$$\begin{aligned} \frac{N!}{(\lceil Nq \rceil - 1)!(N - \lceil Nq \rceil)!} \int_0^p y^{\lceil Nq \rceil - 1} (1-y)^{N - \lceil Nq \rceil} dy = \\ \frac{N!}{\lceil Nq \rceil!(N - \lceil Nq \rceil)!} p^{\lceil Nq \rceil} (1-p)^{N - \lceil Nq \rceil} + \\ \frac{N!}{\lceil Nq \rceil!(N - \lceil Nq \rceil - 1)!} \int_0^p y^{\lceil Nq \rceil} (1-y)^{N - \lceil Nq \rceil - 1} dy. \end{aligned}$$

Notice how the first term on the right-hand side equals the first term in the series S . In the integral on the right-hand side, integrate by parts again setting

$u = (1 - y)^{N - \lceil Nq \rceil - 1}$ and $dv = y^{\lceil Nq \rceil} dy$ to obtain

$$\begin{aligned} \frac{N!}{\lceil Nq \rceil!(N - \lceil Nq \rceil - 1)!} \int_0^p y^{\lceil Nq \rceil} (1 - y)^{N - \lceil Nq \rceil - 1} dy = \\ \frac{N!}{(\lceil Nq \rceil + 1)!(N - \lceil Nq \rceil - 1)!} p^{\lceil Nq \rceil + 1} (1 - p)^{N - \lceil Nq \rceil - 1} + \\ \frac{N!}{(\lceil Nq \rceil + 1)!(N - \lceil Nq \rceil - 2)!} \times \\ \int_0^p y^{\lceil Nq \rceil + 1} (1 - y)^{N - \lceil Nq \rceil - 2} dy. \end{aligned}$$

The first term on the right-hand side equals the second term in the series S . Continue in this fashion until, after the s -th integration by parts with $0 < s < N - \lceil Nq \rceil$, we arrive at the integral

$$\frac{N!}{(\lceil Nq \rceil + s - 1)!(N - \lceil Nq \rceil - s)!} \int_0^p y^{\lceil Nq \rceil + s - 1} (1 - y)^{N - \lceil Nq \rceil - s} dy.$$

Integrate by parts setting $u = (1 - y)^{N - \lceil Nq \rceil - s}$ and $dv = y^{\lceil Nq \rceil + s - 1} dy$ to obtain

$$\begin{aligned} \frac{N!}{(\lceil Nq \rceil + s - 1)!(N - \lceil Nq \rceil - s)!} \int_0^p y^{\lceil Nq \rceil + s - 1} (1 - y)^{N - \lceil Nq \rceil - s} dy = \\ \frac{N!}{(\lceil Nq \rceil + s)!(N - \lceil Nq \rceil - s)!} \\ \times p^{\lceil Nq \rceil + s} (1 - p)^{N - \lceil Nq \rceil - s} \\ + \frac{N!}{(\lceil Nq \rceil + s)!(N - \lceil Nq \rceil - s - 1)!} \\ \times \int_0^p y^{\lceil Nq \rceil + s} (1 - y)^{N - \lceil Nq \rceil - s - 1} dy. \end{aligned}$$

Notice that the first term on the right-hand side equals the $(s + 1)$ -th term in the series S . Continue in this manner until, after performing $(N - \lceil Nq \rceil)$ integrations by parts, we are left with the integral $\frac{N!}{(N-1)!} \int_0^p x^{N-1} dx$, which evaluates to p^N , i.e. the last term in the series S .

A2 - Second Order Conditions

For the investment levels in (5) to be an equilibrium, the second order conditions must be satisfied. The second derivative of the profit function for $i \in \{1, 2\}$ is given

by

$$\frac{\partial^2 \pi_i}{\partial^2 x_i} = v \frac{N!}{\frac{N-1}{2}!^2} p_i^{\frac{N-3}{2}} (1-p_i)^{\frac{N-3}{2}} \left(\frac{N-1}{2} (1-2p_i) \left(\frac{\partial p_i}{\partial x_i} \right)^2 + p_i (1-p_i) \frac{\partial^2 p_i}{\partial^2 x_i} \right) - c_i''(x_i).$$

Substituting the functions p_i and c_i we obtain

$$\frac{\partial^2 \pi_i}{\partial^2 x_i} \propto (1-2p_i) m \frac{N+1}{2} - 1.$$

Therefore, if $m \frac{N+1}{2} \leq 1$, the profit function is concave as desired. If $m \frac{N+1}{2} > 1$, there exists an $\hat{x} > 0$ such that the profit function of candidate i is convex for $x_i < \hat{x}$ and concave for $x_i > \hat{x}$. Consequently, there is a single concave region. This implies that if the profit function is concave at (x_1^*, x_2^*) , then x_1^* is a global maximiser of candidate 1's profit function when candidate 2's investment is x_2^* , and vice versa.

By Equation (4), we have that $p_1 = \frac{1}{1+\theta}$. Thus, the second order conditions are met if and only if

$$\begin{aligned} \left(1 - 2 \frac{1}{1+\theta} \right) m \frac{N+1}{2} - 1 &< 0, \\ \left(1 - 2 \frac{\theta}{1+\theta} \right) m \frac{N+1}{2} - 1 &< 0. \end{aligned}$$

Moreover, for the investment levels in Equation (5) to constitute an equilibrium, both candidates must have non-negative profits. Specifically, given the investment levels in Equation (5), it must be that

$$v \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} p_i^k (1-p_i)^{N-k} - c_i(x_i) \geq 0$$

for both $i \in \{1, 2\}$. Substituting the functions p_i and c_i and using Equation (5) we obtain

$$\begin{aligned} \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} \left(\frac{1}{1+\theta} \right)^k \left(\frac{\theta}{1+\theta} \right)^{N-k} - \frac{N!}{\frac{N-1}{2}!^2} \left(\frac{\theta}{(1+\theta)^2} \right)^{\frac{N+1}{2}} m &\geq 0, \\ \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} \left(\frac{\theta}{1+\theta} \right)^k \left(\frac{1}{1+\theta} \right)^{N-k} - \frac{N!}{\frac{N-1}{2}!^2} \left(\frac{\theta}{(1+\theta)^2} \right)^{\frac{N+1}{2}} m &\geq 0. \end{aligned}$$

A3 - Semi-mixed Strategy Equilibrium: Proofs

We start with the proof of Proposition 6, for which we need a number of auxiliary results.

Lemma 1. *If $\frac{N!}{\frac{N-1}{2}!^2} \frac{1}{2^N} m < 1$ then in the semi-mixed strategy equilibrium we have $\hat{p} > \frac{1}{2}$.*

Proof. From equations (7) and (8) we have that

$$\sum_{k=\frac{N+1}{2}}^N \binom{N}{k} (1-\hat{p})^k \hat{p}^{N-k} = m \frac{N!}{\frac{N-1}{2}!^2} (\hat{p}(1-\hat{p}))^{\frac{N+1}{2}}.$$

Using $\hat{p} = \frac{1}{1+\hat{\theta}}$ we obtain

$$\sum_{k=\frac{N+1}{2}}^N \binom{N}{k} \frac{\hat{\theta}^k}{(1+\hat{\theta})^N} = m \frac{N!}{\frac{N-1}{2}!^2} \frac{\hat{\theta}^{\frac{N+1}{2}}}{(1+\hat{\theta})^{N+1}}.$$

Multiplying both sides by $\frac{(1+\hat{\theta})^{N+1}}{\hat{\theta}^{\frac{N+1}{2}}}$ gives

$$(1+\hat{\theta}) \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} \hat{\theta}^{k-\frac{N+1}{2}} = m \frac{N!}{\frac{N-1}{2}!^2}.$$

The left-hand side above is strictly increasing for $\hat{\theta} > 0$. Moreover, for $\hat{\theta} = 1$, we have

$$\begin{aligned} (1+\hat{\theta}) \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} \hat{\theta}^{k-\frac{N+1}{2}} &= 2 \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} \\ &= 2^N. \end{aligned}$$

Therefore,

$$\hat{\theta} \text{ is } \begin{cases} < 1 & \text{if } \frac{N!}{\frac{N-1}{2}!^2} m < 2^N, \\ = 1 & \text{if } \frac{N!}{\frac{N-1}{2}!^2} m = 2^N, \\ > 1 & \text{if } \frac{N!}{\frac{N-1}{2}!^2} m > 2^N. \end{cases}$$

Since $\hat{p} = \frac{1}{1+\hat{\theta}}$, we have $\hat{p} > \frac{1}{2}$ if and only if $\frac{N!}{\frac{N-1}{2}!^2} \frac{1}{2^N} m < 1$. □

Lemma 2. Define $H_N(\hat{p})$ as

$$H_N(\hat{p}) = \frac{\int_0^{1-\hat{p}} y^{\frac{N-1}{2}} (1-y)^{\frac{N-1}{2}} dy}{(\hat{p}(1-\hat{p}))^{\frac{N+1}{2}}},$$

then, $H_N(\hat{p})$ is decreasing on $\hat{p} > \frac{1}{2}$.

Proof. Differentiate $H_N(\hat{p})$ with respect to \hat{p} to obtain

$$H'_N(\hat{p}) = -\frac{(\hat{p}(1-\hat{p}))^{\frac{N+1}{2}} - \frac{N+1}{2}(2\hat{p}-1) \int_0^{1-\hat{p}} y^{\frac{N-1}{2}} (1-y)^{\frac{N-1}{2}} dy}{(\hat{p}(1-\hat{p}))^{\frac{N+3}{2}}}.$$

Integrating on the second term in the numerator, we get

$$\begin{aligned} \frac{N+1}{2}(2\hat{p}-1) \int_0^{1-\hat{p}} y^{\frac{N-1}{2}} (1-y)^{\frac{N-1}{2}} dy &< \frac{N+1}{2} \int_0^{1-\hat{p}} (1-2y)y^{\frac{N-1}{2}} (1-y)^{\frac{N-1}{2}} dy \\ &< \int_0^{1-\hat{p}} \frac{d}{dy} \left(y^{\frac{N+1}{2}} (1-y)^{\frac{N+1}{2}} \right) dx \\ &< \left[y^{\frac{N+1}{2}} (1-y)^{\frac{N+1}{2}} \right]_0^{1-\hat{p}} \\ &< (\hat{p}(1-\hat{p}))^{\frac{N+1}{2}}. \end{aligned}$$

Hence

$$(\hat{p}(1-\hat{p}))^{\frac{N+1}{2}} - \frac{N+1}{2}(2\hat{p}-1) \int_0^{1-\hat{p}} y^{\frac{N-1}{2}} (1-y)^{\frac{N-1}{2}} dy > 0.$$

Therefore, $H'_N(\hat{p}) < 0$ as desired. \square

Lemma 3. The equilibrium value of δ in the semi-mixed strategy equilibrium decreases with N .

Proof. From equations (7)-(8) and using the Incomplete Beta Function, we must have $H_N(\hat{p}) = m$ (where $H_N(\hat{p})$ is defined in Lemma 2).

Since, by Lemma 1, $\hat{p} > \frac{1}{2}$, we have that

$$\begin{aligned}
H_{N+2}(\hat{p}) &= \frac{\int_0^{1-\hat{p}} y^{\frac{N+1}{2}} (1-y)^{\frac{N+1}{2}} dy}{\hat{p}(1-\hat{p}) (\hat{p}(1-\hat{p}))^{\frac{N+3}{2}}} \\
&< \frac{\hat{p}(1-\hat{p}) \int_0^{1-\hat{p}} y^{\frac{N-1}{2}} (1-y)^{\frac{N-1}{2}} dy}{\hat{p}(1-\hat{p}) (\hat{p}(1-\hat{p}))^{\frac{N+1}{2}}} \\
&< H_N(\hat{p}) \\
&< m.
\end{aligned}$$

Moreover, by Lemma 2, $H_N(\hat{p})$ is decreasing in \hat{p} . Also, note that \hat{p} is increasing in the equilibrium value for δ , call it δ_N when there are N committee members. Therefore, given that in equilibrium $H_{N+2}(\hat{p}) = m$, it must be that $\delta_{N+2} < \delta_N$, i.e. equilibrium δ decreases with N . \square

Lemma 4. *The x_1 that solves (6)-(8) is decreasing in N .*

Proof. Define candidate's 2 probability of winning $W_N(\hat{p})$ as

$$W_N(\hat{p}) = \sum_{k=\frac{N+1}{2}}^N \binom{N}{k} (1-\hat{p})^k \hat{p}^{N-k}.$$

Since $\hat{p} > \frac{1}{2}$, the Condorcet Jury Theorem implies

$$W_{N+2}(\hat{p}) < W_N(\hat{p}). \tag{11}$$

On the other hand, equation (8) implies

$$\alpha_2 x_2 = v W_N(\hat{p}).$$

Since $\frac{x_1}{x_2} = \left(\frac{\hat{p}}{1-\hat{p}}\right)^{\frac{1}{m}}$, we have

$$x_1 = \left(\frac{\hat{p}}{1-\hat{p}}\right)^{\frac{1}{m}} x_2 = \frac{v}{\alpha_2} \left(\frac{\hat{p}}{1-\hat{p}}\right)^{\frac{1}{m}} W_N(\hat{p}).$$

Define the function $F_N(\hat{p})$ as

$$F_N(\hat{p}) = \left(\frac{\hat{p}}{1-\hat{p}}\right)^{\frac{1}{m}} W_N(\hat{p}),$$

with $\hat{p} \in (\frac{1}{2}, 1)$. Then, we have $x_1 = \frac{v}{\alpha_2} F_N(\hat{p})$.

Moreover, since $\left(\frac{\hat{p}}{1-\hat{p}}\right)^{1/m}$ does not depend on N , equation (11) implies

$$F_{N+2}(\hat{p}) < F_N(\hat{p}), \quad (12)$$

for all $\hat{p} \in (\frac{1}{2}, 1)$.

Next we show that the equilibrium \hat{p} is the unique maximizer of F_N . Differentiate $\ln F_N(\hat{p})$ to obtain

$$\frac{\partial}{\partial \hat{p}} \ln F_N(\hat{p}) = \frac{1}{m} \frac{1}{\hat{p}(1-\hat{p})} + \frac{W'_N(\hat{p})}{W_N(\hat{p})}.$$

Using the Incomplete Beta Function, we have

$$W'_N(\hat{p}) = -\frac{N!}{\left(\frac{N-1}{2}\right)!^2} (\hat{p}(1-\hat{p}))^{\frac{N-1}{2}},$$

therefore,

$$\frac{W'_N(\hat{p})}{W_N(\hat{p})} = -\frac{1}{\hat{p}(1-\hat{p})} \frac{1}{H_N(\hat{p})},$$

where $H_N(\hat{p})$ is defined in Lemma 2.

Therefore,

$$\frac{\partial}{\partial \hat{p}} \ln F_N(\hat{p}) = \frac{1}{\hat{p}(1-\hat{p})} \left(\frac{1}{m} - \frac{1}{H_N(\hat{p})} \right). \quad (13)$$

In equilibrium, equations (7)-(8) imply $H_N(\hat{p}) = m$, so by equation (13) the equilibrium \hat{p} is a critical point of F_N . Moreover, by Lemma 2, $H_N(\hat{p})$ is strictly decreasing on $\hat{p} > \frac{1}{2}$, so the right-hand side of equation(13) is positive when $H_N(\hat{p}) > m$ and negative when $H_N(\hat{p}) < m$. Hence, F_N is strictly increasing up to the unique \hat{p} that solves $H_N(\hat{p}) = m$, and strictly decreasing from there on. Thus, \hat{p} is the unique maximizer of F_N on $(\frac{1}{2}, 1)$.

Denote this maximizer by \hat{p}_N . Then, if we let x_{1N} be the equilibrium value of x_1 when committee size is N , we have

$$x_{1,N} = \frac{v}{\alpha_2} \max_{\hat{p} \in (\frac{1}{2}, 1)} F_N(\hat{p}) = \frac{v}{\alpha_2} F_N(\hat{p}_N). \quad (14)$$

Taking maxima over \hat{p} and using equations (12) and (14),

$$x_{1,N+2} = \frac{v}{\alpha_2} \max_{\hat{p}} F_{N+2}(\hat{p}) < \frac{v}{\alpha_2} \max_{\hat{p}} F_N(\hat{p}) = x_{1,N}.$$

Therefore, in the semi-mixed strategy equilibrium the value of x_1 is decreasing in N . □

Proof of Proposition 6. In the semi-mixed equilibrium, we have that

$$\begin{aligned} E(x_1) + E(x_2) &= x_1 + \delta x_2 \\ &= x_1 + \frac{\alpha_1}{\alpha_2} x_1 \\ &= x_1 \left(1 + \frac{\alpha_1}{\alpha_2} \right). \end{aligned}$$

which is decreasing in N since x_1 is decreasing in N by Lemma 4. Moreover, as N increases and the expression $\frac{N!}{N-1!2} \frac{1}{2^N} m$ converges to 1, we have that x_1 converges to $\frac{v}{2\alpha_2}$ by Proposition 3. Therefore, the sum of expected equilibrium investments converges to $\frac{v}{2\alpha_2} \left(1 + \frac{\alpha_1}{\alpha_2} \right)$. □

We conclude this appendix with the proof that the semi-mixed strategy equilibrium, if it exists, is unique.

Lemma 5. *The semi-mixed strategy equilibrium, if it exists, is unique.*

Proof. As argued in the proof of Lemma 3, in equilibrium we must have that $H_N(\hat{p}) = m$. Moreover, by Lemma 1, any semi-mixed equilibrium must then satisfy $\hat{p} > \frac{1}{2}$. On the interval $(\frac{1}{2}, 1)$, the proof of Lemma 3 shows that H_N is strictly decreasing on \hat{p} . Therefore, the equation $H_N(\hat{p}) = m$ has at most one solution $\hat{p} \in (\frac{1}{2}, 1)$. Hence, there is a unique equilibrium value for \hat{p} in the semi-mixed strategy equilibrium.

Moreover, from equations (6)-(8), the equilibrium value of δ depends on \hat{p} through

$$\delta = \frac{\alpha_1}{\alpha_2} \left(\frac{n_2}{n_1} \frac{\hat{p}}{1 - \hat{p}} \right)^{1/m}.$$

Thus, the fact that the equilibrium value of \hat{p} is unique in the semi-mixed strategy equilibrium implies that the equilibrium value of δ is also unique in that equilibrium.

Similarly, from equations (6)-(8) the uniqueness of \hat{p} readily implies the uniqueness of the equilibrium values for x_1 and x_2 . □